METEOROLOGICAL OFFICE
14 OCT 1983
LIBRARY

141466

LIMITED CIRCULATION

This paper is not an official document: it is for information only and must not be quoted in any printed publication

AGRICULTURAL MEMORANDUM NO. 991

Modelling the flow of moisture in a soil

Part II - A Numerical Model to calculate the suction/water content at various depths

by M Field

1. Introduction

Agricultural Memorandum No. 990 (Part 1) deals with the basic concepts of moisture flow in the soil (1). This memorandum describes a numerical model which makes use of the ideas outlined in Part 1. The model is based on the work of Wind and van Doorne (2) with some significant modifications.

2. Summary of the Model

In common with many numerical models a number of equations are solved for each timestep at a number of discrete levels. Some of the equations have analytical solutions but others are solved using iterative techniques. The timestep must be small enough to ensure numerical stability (0.005 day is needed for both sandy loam and clay soils). The model is one-dimensional in the vertical or z-direction. It is run from the 1 September to 31 December each year for up to 20 years using climatological data from the Complete Agromet Dataset (3). The moisture status of the soil on the 1 September each year is found from a simple soil moisture deficit model which runs from 1 January to 31 August. The numerical model calculates the suction/water content of the soil each day during the 122-day autumn period at each of the levels in the soil. The depth of the water table below the surface is also found. The model can be used to estimate the number of Available Work Days for soil-engaging operations in the autumn (see Part III).

3. Governing Equations

The governing equations are,

$$v = -k\left(\partial\psi/\partial z + 1\right) \qquad (3.1)$$

$$k = k_0 e^{\alpha\psi} \qquad (3.2)$$

$$\partial/\partial t\,(\theta_v) = -\partial v/\partial z \qquad (3.3)$$

$$v_b = A z_0 \qquad (3.4)$$

Equation 3.1 is D'arcy's law in 1-dimension (see Part 1) where,

v = flow velocity (cm day^{-1}) measured upwards in the direction of positive z
k = hydraulic conductivity of the soil matrix (cm day^{-1})
ψ = $-s$, where s = suction (cm)

Equation 3.2 is the Conductivity Equation due to Rijtema (4) where,

k_o = saturated hydraulic conductivity of the soil matrix
α = a soil constant (cm^{-1})

Equation 3.3 is the Continuity Equation in 1-dimension where,

θ_v = volumetric moisture content of the soil (cm^3 of water per cm^3 of soil vol.)

This equation can be simply derived from arguments regarding conservation of water, namely that the increase in the amount of water in an element of soil depends on the difference between the flow rates into and out of the element.

Equation 3.4 forms the lower boundary condition to the system and is based on Hooghoudt's equation (see Part 1) where,

z_o = height of water table above the drain (cm)
A = a constant (day^{-1})
v_D = drain discharge velocity (cm day^{-1}) measured positively downwards

Equation 3.4 differs from the one used by Wind and van Doorne (2), see later.

4. Application of Governing Equations to the Unsaturated Zone

The unsaturated zone is the part of the soil above the water table.

Consider the relation $\partial k/\partial z = \partial k/\partial \psi \cdot \partial \psi/\partial z$ (4.1)

where $\partial \psi/\partial z = -v/k - 1$ from 3.1
and $\partial k/\partial z = \alpha k$ from 3.2

Thus $\partial k/\partial z = \alpha k(-v/k - 1)$ or $v + k + 1/\alpha\, \partial k/\partial z = 0$ (4.2)

A solution to this equation can be shown to be

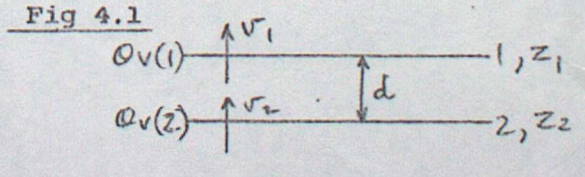
$k + v = Ce^{-\alpha z}$ where C = a constant. (4.3)

Consider 2 discrete levels in the soil

At level 1 $k_1 + v_1 = Ce^{-\alpha z_1}$

At level 2 $k_2 + v_2 = Ce^{-\alpha z_2}$

Fig 4.1



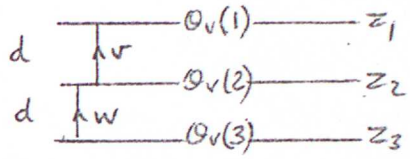
Since we require the velocity to be constant between adjacent levels we let $v_1 = v_2 = v$

Dividing, we find $e^{-\alpha(z_1 - z_2)} = (k_1 + v)/(k_2 + v)$

or $v = \dfrac{k_2 - k_1}{e^{\alpha d} - 1} - k_1$ where $d = z_1 - z_2$ (4.4)

Equation 4.4 enables the flow velocity between any 2 levels to be found from the values of k at the 2 levels.

Consider 3 discrete levels in the soil Fig 4.2



We can apply Equation 3.3 to find the change in Moisture Content θ_v at a given level in the soil from the flow velocities above and below that level.

Equation 3.3 can be expressed in discrete form as

$$\theta'_v(2) = \theta_v(2) + \delta t/d\,(w - v) \quad (4.5)$$

where $\theta_v(2)$ = moisture content at level 2 at time t
$\theta'_v(2)$ = " " " " " " " " $t + \delta t$

Equation 4.5 enables the updated moisture content at a given level to be found from the flow velocities above and below the level.

In brief the model operates thus,

i. Equation 4.4 is used to find the velocities v between the various levels from values of k at the various levels (see Fig 4.3),

ii. Equation 4.5 is used to update the moisture contents θ_v at the various levels from values of v that have just been found,

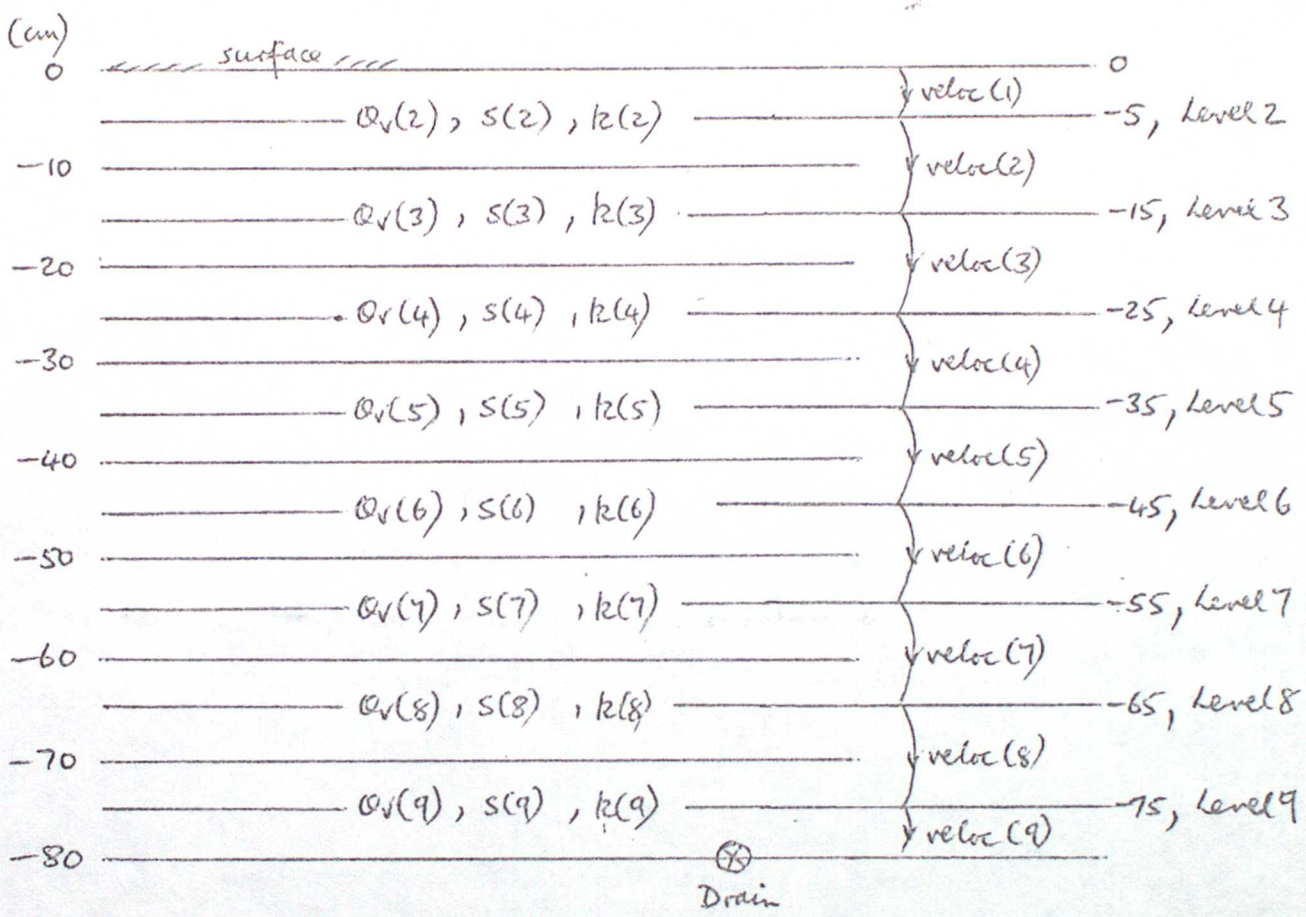
iii. The Soil Moisutre Characteristic (relation between θ_v and s) is used to convert the updated moisture contents θ_v' to suctions s',

iv. Equation 3.2 is used to find new values of k' at the various levels from the values of s' .

The above sequence is repeated starting at i. for each timestep. In this way the velocities, suctions, moisture contents and hydraulic conductivities are found at each level at each timestep.

The discrete levels in the model are as follows,

Fig 4.3 Discrete Levels in the Model

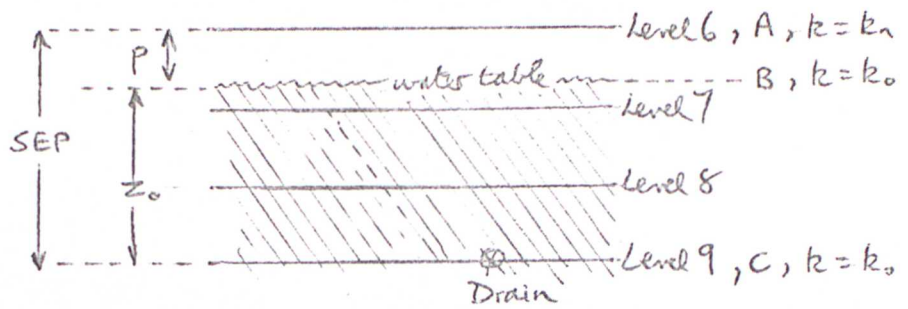


5. Application of Governing Equations to the Saturated Zone

The saturated zone is the part of the soil between the water table and the drains. Consider the situation in which the water table lies between levels 6 and 7.

Fig 5.1

(levels 7-9 are saturated
levels 2-6 are unsaturated)



Between B and C, the saturated zone, we apply Equation 3.4.

Between A and B, the unsaturated zone, we apply Equation 4.4. in which

$k_1 = k_n,\ k_2 = k_o,\ d = \rho \qquad -v_D = (k_o - k_n)/(e^{\alpha\rho} - 1) - k_n$ (5.1)

Equating 3.4 and 5.1 we find

$$Az_o = \frac{k_n e^{\alpha\rho} - k_o}{e^{\alpha\rho} - 1} \tag{5.2}$$

Applying 3.2 and re-arranging we obtain

$$Az_o = k_o/e^{\alpha\rho} - 1\ (e^{\alpha\psi_n} e^{\alpha\rho} - 1) \tag{5.3}$$

Now $A \sim 0.02$ day^{-1} and $z_o \sim 50$ cm, therefore the left-hand-side of 5.3 is of order unity.

For saturated flow along macropores in a well-structured soil, e.g. a subsoiled clay, $k_o \sim 300$ cm day^{-1} (this is about 2 orders of magnitude larger than the saturated hydraulic conductivity for the soil matrix).

Putting $\rho = 5$ cm and $\alpha = 0.3$ cm^{-1} we find in 5.3 that,

$$k_o/e^{\alpha\rho} - 1 \sim 100 \tag{5.4}$$

Thus $e^{\alpha\psi_n} e^{\alpha\rho} - 1 \simeq 0$

or $$\rho \simeq -\psi_n \qquad (5.5)$$

Hence the distance of the water table below the lowest unsaturated level is simply equal to the suction at that level.

Equation 3.4 can be written, $v_D = A(SEP - \rho)$

and from Equation 5.5, $$v_D = A(SEP + \psi_n) \qquad (5.6)$$

Thus the drain discharge velocity as well as the position of the water table can be found from the suction at the lowest unsaturated level.

Wind and van Doorne (2) use a saturated hydraulic conductivity of $k_0 \lesssim 1$ cm day^{-1} which is too small to describe the flow of water to field drains in all but the most poorly structured soils. Thus the above simplifying assumptions do not apply. Their equation to find v_D is more complicated and needs to be solved by iterative techniques. Moreover because their k_0 is much smaller the requirement that $v_D \simeq 1$cm/day for $z_0 = 50$ cm (see Part 1) is not satisified.

Equations 5.5 and 5.6 are invoked each timestep to find the water table position and the drainage discharge velocity. This forms the lower boundary condition.

6. Other Boundary Conditions

6.1 Boundary Conditions on 1 September

In the program it is assumed that there is full ground cover similar to that of cereals up to the end of July during which time a simple procedure calculates SMD in 4 layers of soil. In July the stomatal resistance is increased to allow for senescence of the crop. It is assumed that the crop is harvested on 1 August and that for the remainder of the year the soil is bare. During August a simple SMD balance is used to calculate the deficits under bare soil; the numerical model 'takes over' on the 1 September. The purpose of the calculation of soil moisture deficit up to 1 September is to provide the numerical model with the initial conditions it needs on that date, namely θ_v at the 9 levels of soil (see Fig 4.3).

The cropping pattern built into the model approximates to that of winter barley. However the crop phenology during the summer months is not very critical since the Available Work-Days in the autumn are determined largely by weather conditions in the autumn itself. However antecedent conditions do have some effect, particularly to the field-working in September. The results from the model can be taken to apply more generally to an arable farm in which a crop senesces and is harvested in mid to late Summer.

Ideally the numerical model would begin running before 1 September and would have the facility to allow water to be removed from the soil at various depths by the roots of a transpiring crop, should this be required. Models that can accomplish this do exist but they are complex (5).

6.2 Boundary Conditions at the soil surface after 1 September

6.2.1 Surface Moisture Flux

Since the soil is uncropped during the autumn period the loss of water to the atmosphere is considered to be purely evaporative from the bare surface itself. A simple calculation is performed every day to find the SMD under bare soil. If evaporation > rainfall on any day then the surface moisture flux is negative, i.e. upwards; if evaporation < rainfall it is positive,

e.g.

day n rainfall = 10 mm, bare soil evaporation = 1 mm, surface flux = 9 mm/day

day n+1 rainfall = 0.5mm, bare soil evaporation = 1.6mm, surface flux = -1.1 mm/day

The surface flux is equated to the flow velocity in the top layer of soil, i.e. "veloc (1)" - see Figure 4.3. Thus the upper boundary condition is found.

6.2.2 Infiltration under heavy rainfall

In Wind and van Doorne's model (2) equation 4.4 is used to determine the infiltration velocity when rainfall > evaporation. Under heavy rainfall the maximum velocity permitted by 4.4 can be less than the rainfall rate. This arises because 4.4 is derived using D 'arcy's law in which the saturated hydraulic conductivity is that of the soil matrix, i.e. $\sim$1 cm/day. Normally in a well-structured soil heavy rain will infiltrate completely for example through macropores. Thus in this model rainwater is 'forced' to enter the soil unless the water table is at the surface in which case surface ponding can occur.

7. Moisture Flow under dry conditions

The Rijtema expression (4), see equation 3.2, is not valid for suctions greater than $\sim$ 50 cm. Rijetma recommends the following relation for higher suctions,

$$k = \beta s^{-\omega} \qquad (7.1)$$

where k = hydraulic conductivity of the soil matrix
s = suction
and β and ω are constants

Rijtema found for several different soil types that a value for ω of 1.4 was appropriate. The complete hydraulic conductivity function thus takes the form of equation 3.2 up to a suction of $\sim$ 50 cm and equation 7.1 thereafter. A value for β is found such that the function is continuous.

When calculating the flow velocity between 2 levels equation 4.4. is used provided k at both levels is given by 3.2. This condition is satisfied most of the time. If k at one or both levels is given by 7.1 however then a numerical solution must be found for ν since no analytical solution is available.

Consider equation 4.1, $\partial k/\partial z = \partial k/\partial \psi \cdot \partial \psi/\partial z$

Now $\partial \psi/\partial z = -\nu/k - 1$ (equation (3.1)

and $k = k(\psi)$ or $\psi = \psi(k)$

Thus $\partial k/\partial z$ function $(\nu, k) = M(\nu, k)$

$$\therefore \int_{z_1}^{z_2} dz - \int_{k_1}^{k_2} dk/M(\nu, k) = 0 \qquad (7.2)$$

It can be shown that,

$$M(\nu, k) = 1/[\omega \beta^{-1/\omega} k^{1+1/\omega} (\nu/k + 1)] \qquad (7.3)$$

ν is a constant over the range z_1 to z_2 (see § 4) and so the integrand is a function of k only. The integral can therefore be evaluated numerically.

We need to find the value of ν that satisfies 7.2. A binary chopping method is used. At each iteration the integral is evaluated using a NAG routine.

8. Input data to the model for various soils

8.1 The Soil Moisture Characteristic (S vs θ_v relation)

$S \leq 100$ cm The program is supplied with values of S at evenly-spaced intervals of θ_v (called WCSTEP in the model) between θ_v (S =0) and θ_v (S = 100 cm). When the model is running it converts between θ_v and S by linear interpolation from these fixed points.

e.g. for a peat soil

WCSAT	= θ_v	(S = 0)	= 0.85
WCMIN	= θ_v	(S = 100 cm)	= 0.75
WCSTEP	=	0.025	

The user supplies the corresponding values of S, which are not in general equally-spaced (unlike θ_v values).

e.g. we may have,

S	0	12	40	75	100
θ_v	0.85	0.825	0.80	0.775	0.75

$S > 100$ cm

The user supplies θ_v(PWP = 15000 cm). A curve is fitted between this point and θ_v (s = 100 cm). The curve defines the Soil Moisture Characteristic for $S > 100$ cm.

The equation of this curve is,

$$S = e^{\beta(-m\theta_v + c)} \qquad (8.1)$$

where $m = [\log_{10}(100) - \log_{10}(15000)] / [\theta_v(PWP) - \theta_v(s = 100\,cm)]$

$$c = [\theta_v(s=100cm) \cdot \log_{10}(15000) - \theta_v(PWP) \cdot \log_{10}(100)] / [\theta_v(s=100cm) - \theta_v(PWP)]$$

$$\beta = \ln(10)$$

8.2 Other Input Data

θ_v (S = 50 cm) is supplied to convert values of θ_v in the top level (level 2) to a SMD for output purposes only. Although the concept of SMD is foreign to the numerical model it is of some value to express the moisture content of the top 10 cm layer as a SMD. θ_v (S = 50 cm) defines field capacity or SMD = 0 in this layer.

The other input data that vary with soil-type are

k_o, α, ω and β (see § 3 and § 7)

The value of k at which the 2 sections of the conductivity curve join, k_t, is also required.

8.3 Examples of Input Data for 2 contrasting soils

The 2 constrasting soils are,

i. a well-structured clay, hereafter referred to as the 'heavy' soil,

ii. a sandy loam, hereafter referred to as the 'light' soil.

No single source provided all the require input data. Data are taken from references (4), (6), (7) and (8), see Table 8.3.1 and Figures 8.1 and 8.2.

The behaviour of the 'forced infiltration scheme' is such that the saturated hydraulic conductivity of the matrix, k_o, needs to be greater than the drain discharge velocity, v_D, when the water table

is at the surface i.e. when $V_D = 80 \times 0.02 = 1.6$ cm day^{-1} (see equation 3.4). Thus the model is run for soils in which $k_0 \gtrsim 2$ cm day^{-1}.

It is particularly difficult to find suitable values of k_0 because the method of measurement has a strong bearing on the value obtained, see (9), (10) and (11). In the literature one can find information on the Hydraulic Conducitivity Function of a soil or its Soil Moisture Characteristic but rarely both. It is important to realize however that even if accurate spot values of all the parameters were available they should only be treated as approximate values for a given soil because in general the spatial variability of soil properties is large. For further discussion and quantiative estimates of the means and standard deviations of various soil parameters, see (12).

Table 8.3.1

Soil	$\theta_v(0)$ WCsat	$\theta_v(100cm)$ WCmin	WCstep	$\theta_v(50)$ WCFC	$\theta_v(PWP)$ WCPWP	k_0 cm day^{-1}	α cm^{-1}	B	k_t cm day^{-1}	timestep (day) Δt
Light	0.31	0.26	0.05	0.285	0.12	5.0	0.037	14.2	0.011	0.005
Heavy	0.50	0.45	0.05	0.475	0.30	2.0	0.316	2.96	0.16	0.005

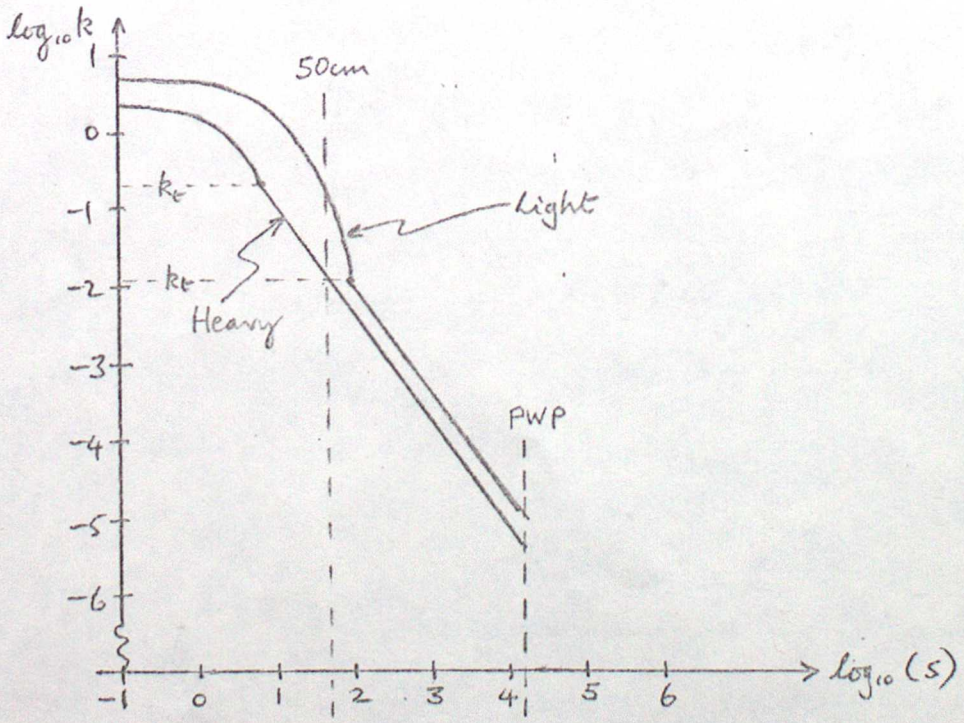
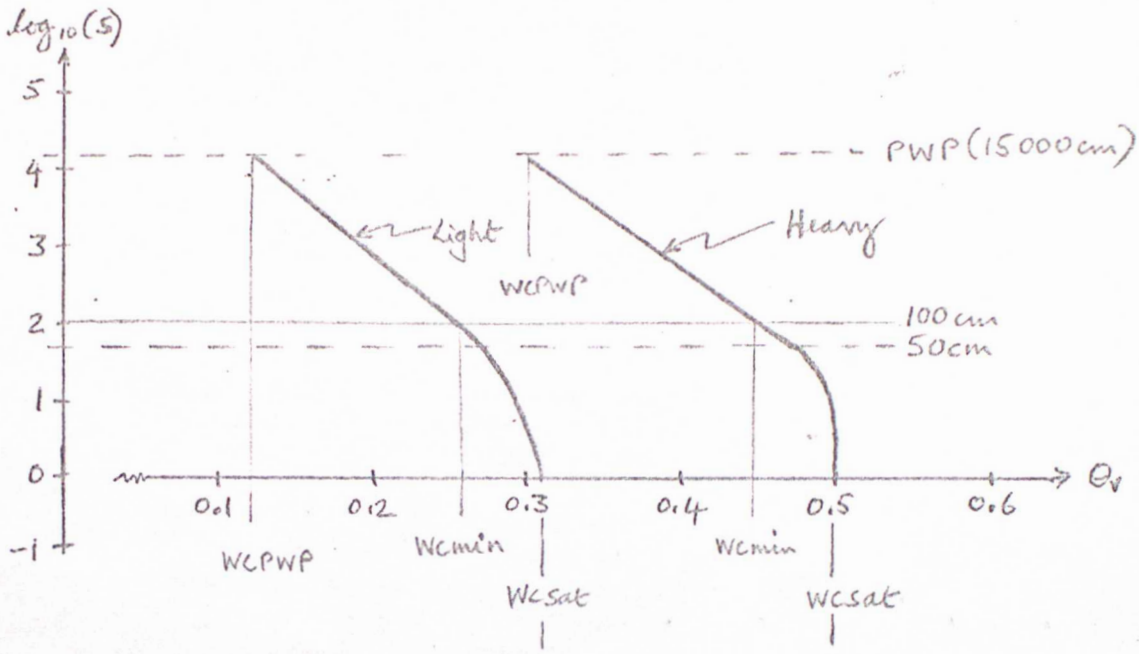


Fig 8.1 Hydraulic Conductivity Function

Fig 8.2 Soil Moisture Characteristic



9. Model Output

Example of graphical output from the model appears in Appendix I.

References

1. Field M. 1983 Agricultural Memorandum 990. Modelling the flow of moisture in a soil. Part 1 - Basic Concepts

2. Wind G.P. and van Doorne W. 1975 A Numerical Model for the Simulation of Unsaturated Vertical Flow of Moisture in Soils. J. Hydrol, 24, 1-20.

3. Field M. 1983 Agricultural Memorandum 983. Creating the 'Complete' Agromet Dataset.

4. Rijtema P.E. 1965 An Analysis of Actual Evaporatranspiration. Institute for Land and and Water Management Research, Wageningen, Netherlands.

5. Belmans C, Wesseling J.G. and Feddes R.A. 1981 Simulation Model of the Water Balance of a cropped soil providing different types of boundary conditions. Instituut voor Cultuurtechniek en Waterhuishouding (ICW) Report No. 1257, Wageningen, Netherlands.

6. Wind G.P. 1979 Analog Modelling of transient Moisture Flow in Unsaturated Soil. Agric. Res. Rep. (Versl. landbouwk. Onderz) 894, Wageningen, Netherlands.

7. MAFF, ADAS 1979 Soil Examination and Management. (A booklet produced by members of Soil Science Dept. of ADAS, Shardlow).

8. Soil Survey 1977 Technical Monograph No. 9. Water Retention Porosity and Density of Field Soils. Harpenden.

9. Feyen J. and Belmans C. 1973 Comparison of five methods for the determination of saturated hydraulic conductivity. Agricultura, 21, 3, 163-183.

10. Shady A.M., Broughton R.S., Bernier Y., Delisle, U. and Silvestre G. 1976 Experience with production scale field measurements of hydraulic conductivity. (Proc. Third National Drainage Symposium, Chicago, USA). Am. Soc. of Ag. Engnrs., 112-116.

11. DeBoer D.W. 1979 Comparison of three field methods for determining saturated hydraulic conductivity. Trans. of ASAE 22 (3), 569-572.

12. Hillel D. 1980 Application of Soil Physics Academic Press. Chapter 13.

Any comments on this paper should be made directly to the author.

M Field
Meteorological Office, Met O 8a
London Road
Bracknell
Berks
RG12 2SZ

10 October 1983

APPENDIX I

'Heavy' soil at Wattisham (Subbolk)